THE

PRECESSION OF EQUINOXES

A NEW ASTRONOMICAL HYPOTHESIS

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THE PRECESSION OF EQUINOXES.

The cause of the difference in the lengths of our solar and sidereal years has been popularly ascribed to the joint attraction of the moon, sun, and planets on the equatorial protuberances of the earth's surface.

It is our purpose in the present treatise to show that the difference of our fundamental periods—the sidereal and mean solar years—is caused by the sun's proper motion, and that the moon and the planets have not the slightest influence on the precession of our equinoxes and the revolution of our polar axis.

Our hypothesis and demonstration are based on the following two laws, which apply to all systems, whether the secondary's revolution round the primary is retrograde or not.

LAW I.

(Showing the relation between the secondary's fundamental periods and the primary's sidereal period.)

The difference of the secondary's fundamental periods (d).

: **the secondary's sidereal period** (p).

: : **the greater of the secondary's fundamental periods** (g).

: **the primary's sidereal period** (P).

In brief form, the Law reads, d : p : : g : P.

LAW II.

(Showing the relation of the secondary's mean orbital velocity to that of the primary.)

The difference of the secondary's fundamental periods (d).

: **the greater of the secondary's fundamental periods** (g).

: : **the primary's mean daily revolution,** expressed in seconds of arc (V).

: **the secondary's mean daily revolution,** expressed in like units (v).

In brief form, the Law reads, d : g : : V : v.

Case I.

When the orbital motion is **retrograde,** the secondary's sidereal period is necessarily the greater of its fundamental periods, and we can substitute p for g, or g for p, in the foregoing proportions.

As, under this Case, d : p : : p : P, we have the following

Rule

(For finding the difference of the secondary's fundamental periods, when the orbital motion is retrograde.)

Divide the square of the secondary's sidereal period by the primary's sidereal period. The quotient expresses the difference of the secondary's fundamental periods. Formula : $d = p^2 \div P$.

Case II.

When the satellite's orbital motion is NOT retrograde, the mean lunar month is the greater of the secondary's fundamental periods.

Rule

(For finding the satellite's greater fundamental period when the sidereal periods of the planet and its moon are both known.)

Divide the planet's sidereal period (P), expressed in days, by the satellite's sidereal period (p). Divide the result, less one unit, into the planet's sidereal period (P). The last quotient obtained expresses the satellite's greater period (g), its mean lunar month. g — p expresses the difference of the secondary's fundamental periods. Formula: $(P \div p) - 1 = r;\ g = P \div r$.

The Rule just given is based on the following algebraic demonstration.

According to Law 1 we have the proportion	$d : p :: g : P$
Let r express the ratio; then	$d : p :: 1 : r$
Substituting g — p for d, and inverting,	$p : g - p :: r : 1$
Therefore	$p = r(g-p)$
But r x g = P. Therefore	$p = P - rp$
Transposing rp and factoring,	$(r + 1)p = P$
And	$p = \dfrac{P}{r+1}$
By hypothesis,	$g = \dfrac{P}{r}$

Hence, the ratio between g and P is one unit less than the ratio between p and P; which agrees with the principles of the Rule above given.

Since we may be the first to announce the foregoing fixed relation of values as Laws applying to every satellite and planet in the solar system, we have expended much time and labor in preparing Tables I, II, III, IV, V, and VI. The calculations there given will serve to show that if our theory is correct with reference to the sun's proper motion being the direct cause of the difference of our fundamental periods, our hypothesis can be verified by bringing the two Laws to bear on the sun's entire planetary system.

The Planets' orbital motion is retrograde.

If the sun has an orbit, be it small or great, the direction of his proper motion is contrary to that of ours, since our sidereal year is the greater of our fundamental periods. In other words, the orbital motion of all the planets—like that of Neptune's moon and Uranus' system of satellites—is **retrograde** with respect to that of their primary.

The varying length of our solar year.

Again, if the sun has an orbital motion, our solar years cannot be exactly the same in length. Our moon's synodical months, for instance, vary in length because there is a difference of over two days in her fundamental periods. If she is near to apogee at the beginning of the month, she revolves at her slowest rate during these two extra days, and the moon's synodical month exceeds the mean length of her lunar period. On the contrary, if at the beginning of the month she is near to perigee, her orbital velocity being fast during the two days in question, the length of her synodical month is shorter than her mean lunar period. But if at the beginning of the month our satellite's orbital position is about midway between apogee and perigee, the moon revolves at her mean velocity during the two days in question and the lunar period in consequence is of average length.

The like principles are involved with respect to the earth's solar year. The difference of our fundamental periods is about .0142 days. When our planet's orbital position at the beginning of the solar year is midway between perihelion and aphelion—in other words, whenever the equinoctial line is perpendicular to the line of apsides—the earth revolves at its mean velocity during the .0142 days. The natural result is a **mean** solar year.

In case the sun has an orbital motion, the correct mean length of our solar year—that is, of the year occurring whenever our equinoctial line is perpendicular to the line of apsides—can be verified by applying the principles enunciated in the first Law. For,

The difference of our fundamental periods (d)
: **our sidereal period** (p)
: : **our sidereal year**—the greater of our fundamental periods (g)
: **the sun's period** (P).

It is evident that if the sun has a proper motion, his period holds the same relation to our sidereal year as the latter holds to the difference of our fundamental periods.

Furthermore, the correct period of the sun can be verified with respect to the entire planetary system by applying the principles enunciated in the second Law; since

The difference of any planet's fundamental periods (d)
: **the planet's sidereal year**—the greater of its fundamental periods (g)

: : **the sun's daily revolution,** expressed in seconds of arc (V)

: **the planet's mean daily revolution,** expressed in like units, (v).

The formula of the proportion is, $v \times d = V \times g$.

Calculation of the Sun's Period.

At the end of our present solar year of $365.24219970416\dot{6}$ days we find the sun 50.37572″ distant from the star with which he was in conjunction the year before. As the alternate angles of two parallel lines are equal, this distance from the star reveals the fact that the sun's orbital motion during our present solar year is 50.37572″. His daily revolution is, therefore, .137924150168853876″. Dividing this number of seconds into 1296000″, the circumference of a circle, we obtain 9396468.9897553821 days, equal to 25725.681987224784 sidereal years, as the sun's period of revolution.

If the sun revolves .137924150168853876″ in one day, in $365.25636\dot{1}$ days, our sidereal period, his orbital motion is 50.377673200018078″, which, according to Law II, must exactly equal the earth's mean revolution during the period expressed by the difference of our fundamental periods.

The difference of our two fundamental periods is computed according to the Rule given under Case I, as follows:

The square of our sidereal year, $365.25636\dot{1}$ days, divided by the sun's period, 9396468.9897553821 days, equals .014198121600527133 days, the required difference.

Multiplying the period expressing the difference by the earth's mean daily revolution of 3548.1928256022007″, we obtain 50.377673200018078″, which, agreeably to the second Law, is **exactly equal** to what we have already found to be the sun's orbital motion during our sidereal year.

The sun's period, as given above, is evidently correct, and our hypothesis true with reference to the sun's proper motion being the cause of the difference in the lengths of our solar and sidereal years. Our calculation can be further verified by employing the sun's period as a factor with respect to the entire planetary system, as shown in Table VII.

TABLE I

Showing that d : p : : g : P, and that d : g : : V : v.

THE EARTH AND ITS SATELLITE, whose orbital motion is NOT RETROGRADE with respect to that of the Primary.

Moon	P *	V †	p Secondary's sidereal period days	v Secondary's mean daily revolution 1296000″ ÷ p seconds of arc	r Primary's period expressed in secondary's mean lunar mos. (P ÷ p) — 1 lunar months	g Satellite's mean lunar period P ÷ r days	d Difference of secondary's fundamental periods g — p days	Secondary's mean revolution in d period d x v seconds of arc	Primary's mean revolution in g period V x g seconds of arc
	P	V	p	v	r	g	d		
Moon	*	†	27.32166134259	47434.8879852449695	12.368746377867642446	29.530588626563857544	2.208927283971264952	104780.222700582926	104780.222700582926

* P = 365.256361 days, the Primary's sidereal period. † V = 3548.1928256022007″, the Primary's mean daily revolution.

TABLE II.

Showing that d : p : : g : P, and that d : g : : V : v.

MARS AND HIS SATELLITES, whose orbital motion is NOT RETROGRADE with respect to that of the Primary.

Moons	P *	V †	p Secondary's sidereal period days	v Secondary's mean daily revolution 1296000″ ÷ p seconds of arc	r Primary's period expressed in secondary's mean lunar mos. (P ÷ p)—1 lunar months	g Satellite's mean lunar period P ÷ r days	d Difference of secondary's fundamental periods g—p days	Secondary's mean revolution in d period d x v seconds of arc	Primary's mean revolution in g period V x g seconds of arc
First	*	†	$.3180\dot{5}$	4074759.82532751 0917	2158.93523144104 80349	.3182028761193796	.0001473205638241	600.295914915	600.295914915
Second	*	†	$1.2597\dot{2}$	1028798.23594266 8136	544.34197133406 83572	1.262036433303787	.002314211081565	2380.85627831	2380.85627831

* P = 686.9794 days, the Primary's sidereal period. † V = 1886.519450219322244″, the Primary's mean daily revolution.

TABLE III.

Showing that d : p : : g : P, and that d : g : : V : v.

JUPITER AND HIS SATELLITES, whose orbital motion is NOT RETROGRADE with respect to that of the Primary.

Name of Satellite	P *	V †	p Secondary's sidereal period days	v Secondary's mean daily revolution 1296000″ ÷ p seconds of arc	r Primary's period expressed in secondary's mean lunar mos. (P ÷ p)—1 lunar months	g Satellite's mean lunar period P ÷ r days	d Difference of secondary's fundamental periods g—p days	Secondary's mean revolution in d period d x v seconds of arc	Primary's mean revolution in g period V x g seconds of arc
Io	*	†	1.76875	732720.848056537	2448.517908127208	1.769472375766225	.000722375766225	529.299784044	529.299784044
Europa	*	†	3.55138̇	364927.649589362	1218.9691263199	3.554302325179	.002913436291	1063.193457	1063.193457
Ganymede	*	†	7.15486̇1	181135.591575269	604.5442212947685	7.166696243859	.0118351327479	2143.7637716	2143.7637716
Callisto	*	†	16.688888̇	77656.458055925	258.608942743009	16.753422190451	.0645333015628	5011.427626	5011.427626

* P = 4332.5848 days, the Primary's siderial period. † V = 299.128594090068358″, the Primary's mean daily revolution.

TABLE IV.

Showing that d : p : : g : P, and that d : g : : V : v.
SATURN AND HIS SATELLITES, whose orbital motion is NOT RETROGRADE with respect to that of the Primary.

Name of Satellite	P *	V †	p Secondary's sidereal period days	v Secondary's mean daily revolution 1296000″ ÷ p seconds of arc	r Primary's period expressed in secondary's mean lunar mos. (P ÷ p)—1 lunar months	g Satellite's mean lunar period P ÷ r days	d Difference of secondary's fundamental periods g—p days	Secondary's mean revolution in d period d x v seconds of arc	Primary's mean revolution in g period V x g seconds of arc
	P	V	p	v	r	g	d		
Mimas	*	†	.9423611111111	1375268.97568165 0700007	11416.3001783935 15117	.9424436563399 84351	.000082545228873 240	113.52189235991	113.52189235991
Enceladus	*	†	1.3699387615740	946027.68850111 39783899	7852.7860847516 5941	1.3701132139279 60377	.000174452353886 303	165.03675710064	165.03675710064
Tethys	*	†	1.88750000	686622.51655629 13907	5699.2488381456 95364	1.8878311839952 25433	.000331183995225 433	227.398388244	227.398388244
Dione	*	†	2.7368055555555	473544.78558741 43618	3930.3058467597 05658	2.7375018895464 10716	.000696333990855 161	329.74533039674	329.74533039674
Rhea	*	†	4.5173611122023 4899	286893.15903907 4726	2380.7488606206 55098	4.5192585660610 5825	.00189745385870920	544.36653165598	544.36653165598
Titan	*	†	15.945593000	81276.37523421 0480601	673.7456605721 718847	15.9692600808186 85113	.023667080818685 113	1923.57454131784	1923.57454131784
Hyperion	*	†	21.2972222222222	60853.00639102 652929073	504.1935679561 7581844	21.3394623926166 0806	.04224017039438584	2570.44135896761	2570.44135896761
Japetus	*	†	79.3298622222222	16336.84924813 7971751	134.6263502873 711169099	79.9191217695016 816	.589259547279459	9626.64439193056	9626.64439193056

* P = 10759.219682 days, the Primary's sidereal period. † V = 120.454832070041936″, the Primary's mean daily revolution.

TABLE V.

Showing that d : p : : g : P, and that d : g : : V : v.

URANUS AND HIS SATELLITES, whose orbital motion is RETROGRADE with respect to that of the Primary.

Name of satellite	p = g Secondary's sidereal period days	v Secondary's mean daily revolution 1296000 ÷ p seconds of arc	P Primary's sidereal period days	V Primary's mean daily revolution 1296000 ÷ P seconds of arc	d Difference of secondary's fundamental periods $p^2 \div P$ days	Secondary's mean revolution in d period v x d seconds of arc	Primary's mean revolution in p period V x g seconds of arc
Ariel	$2.519\dot{4}$	514399.117971510	30686.8205	42.233114375599779	.000206851026115 331	106.403985385	106.403985385
Umbriel	4.14375	312760.180995475	30686.8205	42.233114375599779	.000559545230907 842	175.003467693	175.003467693
Titania	$8.70486\dot{1}$	148882.329477463	30686.8205	42.233114375599779	.002469288304525 865	367.633394929	367.633394929
Oberon	13.4625	96267.409470752	30686.8205	42.233114375599779	.005906082914324 734	568.563302281	568.563302281

TABLE VI.

Showing that d : p :: g : P, and that d : g :: V : v.

NEPTUNE AND HIS SATELLITE, whose orbital motion is RETROGRADE with respect to that of the Primary.

Neptune's moon	p = g Secondary's sidereal period days	v Secondary's mean daily revolution 1296000 ÷ p seconds of arc	P Primary's sidereal period days	V Primary's mean daily revolution 1296000 ÷ P seconds of arc	d Difference of secondary's fundamental periods $p^2 \div P$ days	Secondary's mean revolution in d period v x d seconds of arc	Primary's mean revolution in p period V x g seconds of arc
Satellite	$5.880\dot{5}$	220387.340576	60126.722	21.554476227725037	.000575134189	126.752294	126.752294

TABLE VII.

Showing that d : p : : g : P, and that d : g : : V : v.
THE SUN AND THE PLANETS, whose orbital motion is RETROGRADE with respect to that of the Primary.

Name of Planet	p = g Planet's sidereal period days	v Planet's mean daily revolution 1296000″ ÷ p seconds of arc	P Sun's period.	V Sun's velocity.	d Difference of planet's fundamental periods $p^2 \div P = d$ days	Planet's mean revolution in d period v x d seconds of arc	Sun's revolution in p period V x g seconds of arc
Mercury	87.9692	14732.429077449834715	*	†	.000823562569841616	12.133077151034	12.133077151034
Venus	224.7007	5767.672285845126428	*	†	.005373338073646366	30.991653089846	30.991653089846
Earth	365.256361	3548.1928256022007	*	†	.014198121600527133	50.37767320001 ⎫ 80078 ⎭	50.37767320001 ⎫ 80078 ⎭
Mars	686.9794	1886.519450219322443	*	†	.050225323633686	94.75104992851	94.75104992851
Jupiter	4332.5848	299.128594090068358	*	†	1.997696269700536	597.56807657449	597.56807657449
Saturn	10759.219682	120.454832070042	*	†	12.31960732182975	1483.95623111986	1483.95623111986
Uranus	30686.8205	42.233114375599779	*	†	100.216469976743358	4232.45363884666	4232.45363881666
Neptune	60126.722	21.554476227725037	*	†	384.742683917418956	8292.927034289	8292.927034289

† V, the sun's daily revolution = .137924150168853876″　　　* P, the sun's period = 9396468.9897553821 days.

Cause of the Precession of Equinoxes.

As the equinoctial line and our polar axis revolve 50.37572″ in our solar year, they complete their revolution in the sun's period.

The line revolves by simply maintaining a fixed angle with respect to the radius of the sun's orbit. For,

When the radius of a circle and the tangent to the radius are revolved, the tangent revolves as many seconds of arc as does the radius. And any other line or a series of parallel lines maintaining a fixed angle with respect to the radius, or the radius produced, revolves as many seconds of arc as does the radius.

Conversely, If a line or a series of parallel lines meeting the radius, or the radius produced, constantly revolves as many seconds of arc as does the radius revolving with it, the line or the series of parallel lines maintains a fixed angle with respect to the radius.

The gyration of our polar axis resembles that of a person sitting motionless in a railway coach with body inclined 23½ degrees from an erect position while the car is moving in a circle. By the time the train has covered 50″, or 180°, of the curve, the axis of inclination has described an arc in the heavens equal to 50″, or 180°, as the case may be.

With respect to the occupant of the coach, the line of inclination revolves simply because a fixed angle is maintained with reference to the radius of the train's circular course, while the engine's motion is the direct cause of the line's revolution.

Briefly stated, the precession of equinoxes measures the sun's arc of revolution during our solar year, because the gyration of our polar axis coincides exactly with the sun's orbital motion expressed in angular units.

According to our hypothesis, the **original impulse** imparted by the sun's orbital motion causes our polar axis and with it our equinoctial line to revolve in a given time exactly as many seconds of arc as the sun revolves in his orbit.

The Rotation of the Polar Axis.

If we are correct in our theory that the sun's original impulse causes the rotation of our polar axis, then we have another law that applies to every planet and satellite in the solar system. It is this:

Law III.

The polar axis of every planet and satellite rotates, completing its revolution in the primary's period.

The general confirmation of the law depends on the fact that nature's laws work similarly under like conditions.

If, for instance, the polar axis of our planet completes its revolution in 9396468.9897553821 days because this is the exact period of the sun's revolution, then must the polar axis of Mars and of each of the other planets do the same.

Similarly, if the polar axis of our satellite completes its revolution exactly in 365.256361 days because this is the primary's period of revolution round the sun, then must the original impulse imparted by any planet's orbital motion cause the planet's satellite's polar axis to rotate in the primary's period.

The Libration in Latitude.

Owing to the rotation of the earth's axis in the sun's period, the direction of the axis relative to the sun—in other words, the earth's **libration in latitude** as viewed from the sun—is the same at the beginning and end of every solar year whether the period is reckoned from the winter solstice, the vernal equinox, the summer solstice, or the autumnal equinox.

In like manner, owing to the revolution of the moon's axis in the earth's period, the direction of the axis relative to our planet—in other words, the moon's libration in latitude—is the same at the beginning and end of every lunar month whether the period is reckoned from the date of the full moon or of the new moon, from first quadrature or from last quadrature.*

The libration in latitude observed when we compare one phase of our satellite with another—for instance, the moon in first quadrature with the phase two weeks afterward—is because the axis describes a comparatively small arc in that time, remaining in a limited sense "parallel to itself."

Precisely the same condition of things exists with reference to the rotation of the earth's polar axis. At the end of six months our planet's libration in latitude, as viewed from the sun, is considerable. At the summer solstice, for example, the north pole is turned about 23½ degrees towards the sun; at the winter solstice it is turned as many degrees away from it, because in six months' time our axis revolves only 25.18786″, remaining very nearly parallel to itself.

The explanation in both cases is the same.

The Variation in the Obliquity of the Ecliptic.

The secular variation in the obliquity of the ecliptic has been attributed to the joint attraction of Mars, Jupiter, and Venus (the sun and moon being adjudged without influence in this case) on the mass of the earth.

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*If our satellite's axis did NOT rotate in the earth's period and ever pointed to the same fixed star, the sixth full moon after the present one would show the libration we notice about five days before our satellite is in conjunction with the sun. Periodically, we would see the full moon rise with the face inclined, as though "the man in the moon" were resting his head on a pillow.

According to our hypothesis, not only the moon, but our nearest planets also, are innocent of any appreciable influence. The simple explanation is, that the plane of the sun's orbit, instead of coinciding with the plane of the earth's orbit, is somewhat inclined to it.

On the supposition that the apparent arc of inclination measures $53.29725'$, the maximum variation in the obliquity of the ecliptic is twice $53.29725' = 6395.67''$.

As this variation is effected in 12863.34 solar years, the greatest possible variation during any one year is $.4972''$.

If the sun has a proper motion, a close relation must necessarily exist between the apparent arc of inclination of the plane of his orbit and the maximum degree of variation. After one of the values has been accurately ascertained the other value can readily be found.

We conclude, therefore, that the secular variation in the obliquity of the ecliptic is due to the inclination of the plane of the sun's orbit to the plane of the earth's orbit.

The Glacial Epoch.

If no misfortune befalls the earth at the end of any solar year in the present era of the world on account of the rotation of our polar axis, no catastrophe need be anticipated by future generations at the end of 12863 solar years, by which time the axis will have described an arc in the heavens of 180°. The sun that year will be about 23° north of the equator at the summer solstice and south of it in winter, as now.

What, then, becomes of the theory regarding the earth's glacial period? It is an undeniable fact that our planet has sustained at least one; and our neighbor Mars has had one more than the earth. But mankind need not fear the recurrence of another so long as the king of day wields his flaming scepter in the sky—unless, alas, another planet-ring should form round the equatorial regions of our luminary.

Imagine a red-hot stove in an open field during the bleak days of winter, with a sheet of zinc surrounding the cylinder and shutting off much of the heat that would radiate but for the interposing belt of zinc direct to those standing round about, and no vivid imagination is necessary to paint a mental picture of what happened in a bygone age when a planet-ring absorbed or intercepted much of the warmth that would have reached the earth but for the interposing medium.

Gradually the ring got denser as the sun contracted and the distance between ring and sun increased. Little by little our summers grew shorter and the time came when the temperature sank to that of autumn. After a while winter reigned supreme most of the year in the temperate zones. Dense clouds hid the azure vault of heaven, and no stars sparkled in the firmament. Cloudland absorbed much of the warmth that would have reached the frozen earth, and sent down sleet and snow, snow and sleet for ages. Cold winds swept the earth and leveled the sapless forests. Except possibly in the torrid zone, vegetation became a thing of the past; for where were the springs, the brooklets, the rivers?

North and south of the tropical region all was still, for animal life could not exist. The last mastodon had long since lain down to rise no more. The last feathered songster of the forest had long since chanted its own requiem. Valleys once fragrant with the breath of flowers were then covered by deep seas of ice and snow. It is a dismal picture, and we are not loath to draw away from it.

But when the obstructing planet-ring parted at last and the atoms composing it formed a new-born sphere! Then came forth the king of day in full majesty and splendor, shining gloriously as of yore! All nature felt the touch of his warm, invigorating hand. The snows first responded to what was expected of them. They gradually melted away, deluging the lowlands. Vast fields of ice glided to lower levels bearing on their surface rocks which intermittent frosts had torn from the mountainside. Objecting, as it were, to being borne far away from their native homes, some of the rocks engraved their protest on those beneath in characters that are read with interest by geologists of the present day.

The glacial epoch having lasted through ages, it took many a year to restore the world to its pristine beauty. Our planet was not created six thousand years ago, but its rejuvenescence dates from the time when the glacial period was relegated—not to oblivion, but—to things of bygone times.

Mars shared in the happy rejuvenescence of the new epoch. More than one world rejoiced at the birth of the infant planet! Can Mars, it may be asked, boast of an older civilization than ours, and, supposing that both of our nearest planets are inhabited, can we truthfully affirm that our civilization antedates that of Venus?

The linear Dimensions of the Sun's Orbit.

Our calculations serve to show that the precession of the equinoxes indicates the number of seconds of arc revolved by the sun during our solar year; but the arc of his revolution as seen from our planet is a very different matter.

While we may feel confident that our luminary's arc of revolution, as measured from his center of motion, is 50.37572″ at the end of our solar year, and that the daily arc is .1379241501068853876″, we are at present in ignorance as to the true dimension of either arc as seen from our planet.

Since the data are still wanting for instituting a comparison between the size of the earth's orbit and that of the sun, his daily revolution, expressed in linear units, must for the present remain an unknown quantity.

The indications are, however, that his center of motion is located within his mass. If this supposition should prove correct, his daily velocity, expressed in linear units, may not exceed .005 miles and the radius of his orbit 7478 miles.

Let X represent the sun's daily revolution, expressed in miles;
P, his period in days;
O, his orbit, expressed in miles;
p, the sidereal period of any planet, expressed in days;
d, the difference of any planet's fundamental periods; and
v, any planet's mean daily velocity, expressed in seconds of arc, or miles.

Since we reckon only by **mean** distances, **mean** velocities, and **mean** periods,

C stands for any planet's orbit or the circumference of a circle.

Therefore, $P \times X =$ the sun's orbit (O); and $p \times v =$ any planet's orbit (C).

The orbital motion of all the planets being retrograde with respect to that of the sun, we have, agreeably to Law I, the following proportion:

$$d : p :: p : P$$

Multiplying the first two terms by v, and the last two by X, we obtain $d \times v : p \times v :: p \times X : P \times X$.

But $p \times v =$ any planet's orbit (C), and

$P \times X =$ the sun's orbit (O).

Hence, $$d \times v : C :: p \times X : O$$

That is,

The difference, expressed in days, of any planet's fundamental periods, multiplied by its mean daily revolution, expressed in seconds of arc or miles,

: the planet's orbit, expressed in like units,

: : the planet's sidereal period, multiplied by the sun's daily revolution, expressed in miles,

: the sun's orbit, expressed in miles.

The following Table illustrates the truth of this proportion and shows, also, that the astronomers of any inhabited planet would agree with reference to the dimensions of the sun's orbit, if the primary's daily velocity, expressed in linear units, were a known quantity and they went on the theory that our luminary's period is 9396468.9897553821 days.

TABLE VIII.

Showing that d x v : C : : p x X : O; and that the Sun's period is 9396468.98975538214 days.

THE RELATION OF THE PLANETS' VELOCITIES AND FUNDAMENTAL PERIODS TO THE SUN'S ORBIT.

Name of Planet.	d x v	:	C	::	p x X			:	O	The Ratio, whatever the value of X may be.
	Planet's mean daily Revolution, multiplied by the difference of the fundamental periods.		Planet's Orbit.		Sun's Revolution, expressed in miles, during the planet's sidereal period.				Sun's Orbit.* The assumed value of X taken as .005 miles.	
	Seconds of arc.		Seconds of arc.		Planet's period x X				9396468.98975538214 days times X. Miles.	O ÷ (p x X)
	d x v		p x v		p x X				P x X	C ÷ (d x v)
Mercury	12.133077151033940388	:	1296000	::	87.9692	x	X	:	46982.3449487769107	106815.4421065029822
Venus	30.9916530898465841	:	1296000	::	224.7007	x	X	:	46982.3449487769107	41817.7112476969682
Earth	50.3776732000180078	:	1296000	::	365.256361	x	X	:	46982.3449487769107	25725.68198722478387
Mars	94.7510499285091344 22	:	1296000	::	686.9794	x	X	:	46982.3449487769107	13677.9486979600584
Jupiter	597.568076574493737	:	1296000	::	4332.5848	x	X	:	46982.3449487769107	2168.790554256521912
Saturn	1483.956231119856246	:	1296000	::	10759.219682	x	X	:	46982.3449487769107	873.341122077423732
Uranus	4232.4536388466635835	:	1296000	::	30686.8205	x	X	:	46982.3449487769107	306.2053623233916379
Neptune	8292.927034288930061	:	1296000	::	60126.722	x	X	:	46982.3449487769107	156.2777526730191967

*The dimension of the Sun's orbit must change uniformly, as the assumed value of X is decreased or increased.

SUMMARY.

First Law.

The difference of the secondary's fundamental periods
: the secondary's sidereal period
: : the greater of the secondary's fundamental periods
: the primary's sidereal period.

Second Law.

The difference of the secondary's fundamental periods
: the greater of the secondary's fundamental periods
: : the primary's mean daily revolution, expressed in angular units,
: the secondary's mean daily revolution, expressed in like units.

Case I.

When the orbital motion is **retrograde,** the secondary's sidereal period is the greater of its fundamental periods.

Case II.

When the orbital motion is **not retrograde,** the satellite's lunar period is the greater of its fundamental periods.

Third Law.

The polar axis of every planet and satellite rotates, completing its revolution in the primary's period.

Hypothesis.

1. The sun has a proper motion, whose direction is contrary to that of the planets.

2. His period is 9396468.9897553821 days, and his daily revolution is .137924150168853876 seconds of arc.

3. The sun is a fixed star, since his orbit is comparatively small.

4. Our mean solar year occurs whenever the equinoctial line is perpendicular to the line of apsides.

5. The original impulse imparted by the sun's proper motion causes the precession of equinoxes.

6. The original impulse imparted by the earth's orbital motion causes the rotation of the moon's polar axis.

7. The original impulse imparted by any primary's orbital motion causes the secondary's polar axis to rotate in the primary's period.

8. The secular variation in the obliquity of the ecliptic is due to the sun's proper motion, in an orbit whose plane is inclined to the plane of the ecliptic.

9. The age immediately preceding the formation of the newest planet in the solar system, and our glacial period, were cotemporaneous epochs.

First Rule.

To find the mean solar year of any planet.

Divide the square of the planet's sidereal year, expressed in days, by the sun's period, expressed in days. Subtract the quotient from the planet's sidereal period. The result obtained is the planet's mean solar year expressed in days.

Second Rule.

To find the mean lunar month of a satellite whose orbital motion is retrograde.

Divide the square of the satellite's sidereal month, expressed in days, by the planet's sidereal period, expressed in days. Subtract the quotient from the satellite's sidereal period. The result obtained is the satellite's mean lunar month.

Third Rule.

To find the mean lunar month of a satellite whose orbital motion is NOT retrograde.

Divide the satellite's sidereal period, expressed in days, into the planet's sidereal period, expressed in days. From the quotient obtained subtract **one unit**. Divide the remainder into the planet's sidereal period. The result is the satellite's mean lunar month, expressed in days.

Argument.

We have endeavored to prove that the sun has a proper motion just as an astronomer on Ariel, one of Uranus' four satellites—whose orbital motion, like that of the planets, is retrograde with respect to that of their primary—might essay to demonstrate that Uranus has an orbit.

Ariel's sidereal period is $2.519\dot{4}$ days. If one of Ariel's astronomers were to take his observation at a time when the planet's orbital position is midway between aphelion and perihelion—that is, when Uranus revolves at his mean velocity of 42.233114375599779″ per day—the astronomer would find at the end of a lunar month of, for instance, 2 days, 12 hours, 27 minutes, and 38.7537 seconds, that the primary is 106.39359999841″ distant from the star with which he was in conjunction when the lunar month began.

Supposing that Uranus were self-luminous and the sun invisible, Ariel's astronomer would be placed in a position analogous to ours in at-

tempting to prove to himself and others that the difference between Ariel's lunar and sidereal months is due, not to Uranus' or to any of the three other satellites' attraction on the equatorial protuberances of Ariel's surface, but to the planet's proper motion through space at the mean daily velocity of $42.233114375599779''$, which is the quotient obtained by dividing Uranus' distance from the fixed star by $2.519198538\dot{1}94$ days, the length of Ariel's lunar month at the time.

Dividing the primary's mean daily velocity thus obtained into the circumference of a circle, expressed in seconds of arc, Ariel's astronomer would obtain 30686.8205 days as Uranus' period of revolution.

He would then verify this period as shown in Table V and as we have verified the sun's in Table VII, by the orbital motion of the secondary planets in the entire system.

The demonstration submitted for their inspection and careful examination ought, it seems to us, convince Ariel's astronomers that Uranus has a proper motion, even though the sun were invisible and the true location of the center of motion undetermined.

Ariel's astronomer would explain furthermore, that the secular variation in the obliquity of Uranus' apparent course through the heavens—the reflex of the satellite's monthly revolution round the primary—shows that the primary has an orbit whose plane is inclined to the plane of Ariel's orbit.

In like manner, we have tried to show that the sun has an orbit whose plane is inclined to that of the ecliptic, and that the difference of our fundamental periods and the precession of equinoxes are due to the sun's proper motion.